Question	Scheme	Marks	AOs
1(a)	Angle $AOB = \dfrac{\pi-\theta}{2}$	B1	2.2a
		(1)	
(b)	Area = $2\times\frac{1}{2}r^2\left(\frac{\pi-\theta}{2}\right)+\frac{1}{2}(2r)^2\theta$	M1	2.1
	$=\frac{1}{2}r^2\pi-\frac{1}{2}r^2\theta+2r^2\theta=\frac{3}{2}r^2\theta+\frac{1}{2}r^2\pi=\frac{1}{2}r^2(3\theta+\pi)$ *	A1*	1.1b
		(2)	
(c)	Perimeter = $4r+2r\left(\frac{\pi-\theta}{2}\right)+2r\theta$	M1	3.1a
	$=4r+r\pi+r\theta$ or e.g. $r(4+\pi+\theta)$	A1	1.1b
		(2)	
			(5 marks)

Notes

(a)

B1: Deduces the correct expression for angle AOB

Note that $\frac{180-\theta}{2}$ scores B0

(b)

M1: Fully correct strategy for the area using their angle from (a) appropriately.

Need to see $2\times\frac{1}{2}r^2\alpha$ or just $r^2\alpha$ where α is their angle in terms of θ from part (a) + $\frac{1}{2}(2r)^2\theta$ with or without the brackets.

A1*: Correct proof. For this mark you can condone the omission of the brackets in $\frac{1}{2}(2r)^2\theta$ as long as they are recovered in subsequent work e.g. when this term becomes $2r^2\theta$

The first term must be seen expanded as e.g. $\frac{1}{2}r^2\pi-\frac{1}{2}r^2\theta$ or equivalent

(c)

M1: Fully correct strategy for the perimeter using their angle from (a) appropriately

Need to see $4r+2r\alpha+2r\theta$ where α is their angle from part (a) in terms of θ

A1: Correct simplified expression

Note that some candidates may change the angle to degrees at the start and all marks are available e.g.

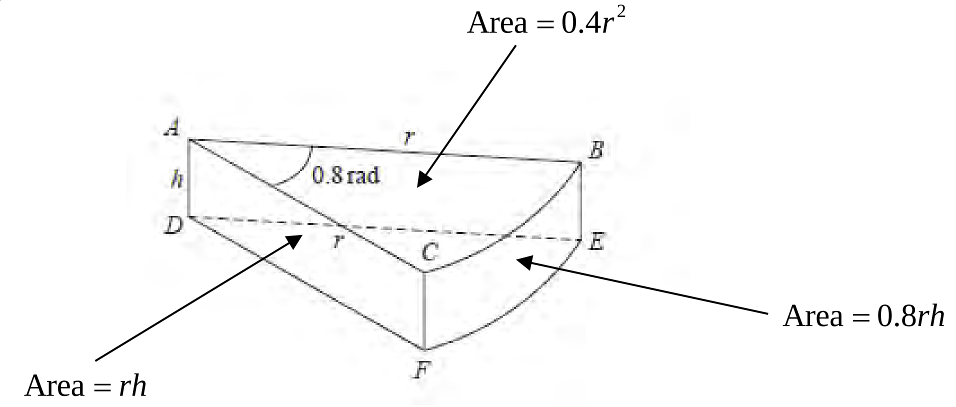
(a) $\dfrac{180-\frac{180\theta}{\pi}}{2}$

(b) $2\left(\dfrac{180-\frac{180\theta}{\pi}}{2}\right)\times\frac{1}{360}\times\pi r^2+\frac{\theta}{360}\times\frac{180}{\pi}\times\pi(2r)^2=\frac{1}{2}\pi r^2-\frac{1}{2}r^2\theta+2r^2\theta=\frac{1}{2}r^2(3\theta+\pi)$

(c) $4r+2\left(\dfrac{180-\frac{180\theta}{\pi}}{2}\right)\times\frac{1}{360}\times2\pi r+\frac{180\theta}{\pi}\times\frac{1}{360}\times2\pi(2r)=4r+\pi r+r\theta$

Question	Scheme	Marks	AOs
2 (a)	Sets up an allowable equation using volume $= 240$ E.g. $\frac{1}{2}r^2 \times 0.8h = 240 \Rightarrow h = \frac{600}{r^2}$ o.e.	M1 A1	3.4 1.1b
	Attempts to substitute their $h = \frac{600}{r^2}$ into $(S =)\frac{1}{2}r^2 \times 0.8 + \frac{1}{2}r^2 \times 0.8 + 2rh + 0.8rh$	dM1	3.4
	$S = 0.8r^2 + 2.8rh = 0.8r^2 + 2.8 \times \frac{600}{r} = 0.8r^2 + \frac{1680}{r}$ *	A1*	2.1
		(4)	
(b)	$\left(\frac{dS}{dr}\right) = 1.6r - \frac{1680}{r^2}$	M1 A1	3.1a 1.1b
	Sets $\frac{dS}{dr} = 0 \Rightarrow r^3 = 1050$ $r = \text{awrt } 10.2$	dM1 A1	2.1 1.1b
		(4)	
(c)	Attempts to substitute their positive r into $\left(\frac{d^2S}{dr^2}\right) = 1.6 + \frac{3360}{r^3}$ and considers its value or sign	M1	1.1b
	E.g. Correct $\frac{d^2S}{dr^2} = 1.6 + \frac{3360}{r^3}$ with $\left.\frac{d^2S}{dr^2}\right\|_{r=10.2} = 5 > 0$ proving a minimum value of S	A1	1.1b
		(2)	
			(10 marks)
Notes:			

Volume $= 0.4r^2h$



Total surface area = $2rh + 0.8r^2 + 0.8rh$

(a)

M1: Attempts to use the fact that the volume of the toy is 240 cm^3

Sight of $\frac{1}{2}r^2 \times 0.8 \times h = 240$ leading to $h = ...$ or $rh = ...$ scores this mark

But condone an equation of the correct form so allow for $kr^2h = 240 \Rightarrow h = ...$ or $rh = ...$

A1: A correct expression for $h = \frac{600}{r^2}$ or $rh = \frac{600}{r}$ which may be left unsimplified.

This may be implied when you see an expression for S or part of S E.g $2rh = 2r \times \frac{600}{r^2}$

dM1: Attempts to substitute their $h = \frac{a}{r^2}$ o.e. such as $hr = \frac{a}{r}$ into a **correct** expression for S

Sight of $\frac{1}{2}r^2 \times 0.8 + \frac{1}{2}r^2 \times 0.8 + rh + rh + 0.8rh$ with an appropriate substitution

Simplified versions such as $0.8r^2 + 2rh + 0.8rh$ used with an appropriate substitution is fine.

A1*: Correct work leading to the given result.

$S =$, $SA =$ or surface area = must be seen at least once in the correct place

The method must be made clear so expect to see evidence. For example

$S = 0.8r^2 + 2rh + 0.8rh \Rightarrow S = 0.8r^2 + 2r \times \frac{600}{r^2} + 0.8r \times \frac{600}{r^2} \Rightarrow S = 0.8r^2 + \frac{1680}{r}$ would be fine.

(b) There is no requirement to see $\frac{dS}{dr}$ in part (b). It may even be called $\frac{dy}{dx}$.

M1: Achieves a derivative of the form $pr \pm \frac{q}{r^2}$ where p and q are non- zero constants

A1: Achieves $\left(\frac{dS}{dr}\right) = 1.6r - \frac{1680}{r^2}$

dM1: Sets or implies that their $\frac{dS}{dr} = 0$ and proceeds to $mr^3 = n,\quad m \times n > 0$. It is dependent upon a correct attempt at differentiation. This mark may be implied by a correct answer to their $pr - \frac{q}{r^2} = 0$

A1: $r =$ awrt 10.2 or $\sqrt[3]{1050}$

(c)

M1: Attempts to substitute their positive r (found in (b)) into $\left(\frac{d^2S}{dr^2} =\right) e \pm \frac{f}{r^3}$ where e and f are non zero and finds its value or sign.

Alternatively considers the sign of $\left(\frac{d^2S}{dr^2} =\right) e \pm \frac{f}{r^3}$ (at their positive r found in (b))

Condone the $\frac{d^2S}{dr^2}$ to be $\frac{d^2y}{dx^2}$ or being absent, but only for this mark.

A1: States that $\frac{d^2S}{dr^2}$ or $S'' = 1.6 + \frac{3360}{r^3} =$ awrt $5 > 0$ proving a minimum value of S

This is dependent upon having achieved $r =$ awrt 10 and a correct $\frac{d^2S}{dr^2} = 1.6 + \frac{3360}{r^3}$

It can be argued without finding the value of $\frac{d^2S}{dr^2}$. E.g. $\frac{d^2S}{dr^2} = 1.6 + \frac{3360}{r^3} > 0$ as $\underline{r > 0}$, so minimum value of S. For consistency it is also dependent upon having achieved $r =$ awrt 10

Do **NOT** allow $\frac{d^2y}{dx^2}$ for this mark

Question	Scheme	Marks	AOs
3(a)	$OC \times 2.3 = 27.6$	M1	1.1b
	e.g. $OC = \frac{27.6}{2.3} = 12\text{ m}$ *	A1*	2.1
		(2)	
(b)	e.g. $(2AOB =)\ \pi - 2.3$	M1	1.1b
	$\frac{\pi - 2.3}{2} \Rightarrow 0.421$ rad *	A1*	2.1
		(2)	
(c)	Area $OCDE = \frac{1}{2} \times 12^2 \times 2.3$	M1	1.1b
	$= 165.6\ (\text{m}^2)$ (accept awrt 166)	A1	1.1b
	$(OB =)\ \frac{35 - 27.6}{2} + 12 = 15.7\text{ m}$	B1	2.1
	Area of OAB (or OFG) $= \frac{1}{2} \times "15.7" \times 7.5 \times \sin 0.421 \quad (= 24.0...\text{m}^2)$	M1	1.1b
	Total area $= 165.6 + 2 \times "24.1"$	dM1	3.1a
	$=$ awrt $214\ (\text{m}^2)$	A1	1.1b
		(6)	
			(10 marks)

Notes

(a)

M1: Uses $l = r\theta$ with $l = 27.6$ and $\theta = 2.3$ correctly substituted in (may be labelled differently in their equation). Values just need to be embedded in an equation or accept an expression for OC e.g. $\frac{27.6}{2.3}$.
May work in degrees which is acceptable.
Condone an alternative letter being used to denote OC such as r
Alternatively, they use $l = r\theta$ with $r = 12$ and $\theta = 2.3$ and verify that $l = 27.6\text{ m}$

A1*: Achieves an expression for OC before proceeding to $OC = 12$ (m) with no errors seen (condone lack of units)
They must show at least $\frac{27.6}{2.3} \Rightarrow OC = 12$ (m) which can score M1A1*
$r = \frac{27.6}{2.3} = 12$ is M1A1* (condone alternative letters for OC)
BUT e.g. $\frac{27.6}{2.3} = 12$(m) on its own is M1A0*
e.g. $OC \times 2.3 = 27.6 \Rightarrow OC = 12$(m) is M1A0*

In the alternative method they verify $l = 27.6$ and conclude that $OC = 12\text{ m}$
We must see the calculation $12 \times 2.3 = 27.6$ and conclude that $OC = 12$ (m)
e.g. arc $= 12 \times 2.3 = 27.6$ so $OC = 12$(m) is M1A1* whereas $12 \times 2.3 = 27.6$ is M1A0*
Also allow e.g. if $OC = 12$(m) then $12 \times 2.3 = 27.6$ ✓ is M1A1*
If they work in degrees and use rounded values this scores A0* (If they work with e.g. $\frac{414}{\pi}$ to keep the angle exact then A1* can still be scored)

(b)

M1: Attempts to subtract 2.3 from π (which may be implied by an expression for *AOB* which is not the given answer)

e.g. $\frac{1}{2}(\pi - 2.3)$ or $\frac{\pi}{2} - 1.15$ score M1

May work in degrees e.g. $180 - \text{awrt}132$ is M1

Condone invisible brackets e.g. $\pi - 2.3 \div 2$ can still score M1.

A1*: Achieves 0.421 (rad) with no errors seen (ignore any side working which is not part of their main solution). Look for a correct expression which is awrt 0.421 before proceeding to the answer. Alternatively, they may write

e.g $2AOB = \pi - 2.3\ (= 0.8415..) \Rightarrow AOB = 0.421$

Condone if they do not round their answer at the end to 0.421.

Condone lack of units. Condone poor labelling of other angles and it does not require *AOB* = to score this mark, but do not accept e.g. *ABO* =

If they work in degrees then withhold this mark if they do not show the conversion back to radians.

e.g. $\frac{\pi - 2.3}{2} = 0.421(\text{rad})$ is M1A1*

e.g. $\frac{180 - \text{awrt}131.8}{2} \div \frac{180}{\pi} = 0.421$ (rad) is M1A1* (conversion from degrees to radians seen)

e.g. $\pi - 2.3 \div 2 = 0.421$ (rad) is M1A0* (invisible/lack of brackets)

e.g. $\pi - 2.3 = \frac{0.842...}{2} = 0.421$ M1A0* (incorrect joined statement)

(c)

M1: Attempts to use $A = \frac{1}{2}r^2\theta$ with $r = 12$ and $\theta = 2.3$ The values embedded in the formula is sufficient for this mark. May be implied by a correct answer or further work. Look out for alternative more complex ways to find the area of the sector. e.g. area of semicircle – area of two sectors with $r = 12$ and $\theta = 0.421$

A1: awrt 166 (may be implied by later work)

B1: A correct expression or value for the length *OB* or *OF* which may be a part of a calculation (may see 15.7 in the equation to find the area of *AOB*)

M1: Attempts to find the area of at least one of the two congruent triangles using their *OB* found from $\frac{35 - 27.6}{2} + 12\ (= 15.7)$, $OA = 7.5$ and $\theta = 0.421$ in $\frac{1}{2} \times OA \times OB \times \sin C$ (may work in degrees)

Be aware that omitting sine in the formula may give a value close to the area of the triangle which would be M0. **Condone use of $\theta = 0.4$ or $\theta = 0.42$** if they have rounded angle *AOB*.

The values embedded in the expression is sufficient to score the mark or may be implied by the value.

Look out for more complex methods to find the area of one or both of the two congruent triangles

e.g. they may split the congruent triangle into two right angled triangles and add the separate areas.

Other alternatives

e.g. finding the area of the trapezium *ABFG*:

$BF = 2\times 15.7\cos 0.421$

Area of $AOB = \frac{1}{2}\left(\left(\frac{15+2\times 15.7\cos 0.421}{2}\right)\times 15.7\sin 0.421 - \frac{1}{2}\times 15.7^2\times \sin 2.3\right)$ o.e.

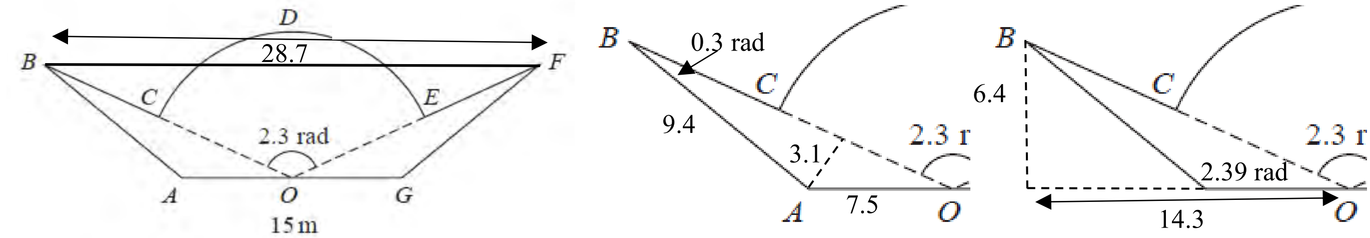
e.g. finding the length *AB* and either angle *OAB* or angle *ABO*:

$AB^2 = 15.7^2 + 7.5^2 - 2\times 15.7\times 7.5\times \cos 0.421 \Rightarrow AB = 9.37...$

$\frac{\sin ABO}{7.5} = \frac{\sin 0.421}{9.37...} \Rightarrow ABO = 0.333...$ or $\frac{\sin OAB}{15.7} = \frac{\sin 0.421}{9.37...} \Rightarrow OAB = 2.4$

Area of ABO $\frac{1}{2}\times 15.7\times 9.37...\times \sin 0.333...$ or $\frac{1}{2}\times 7.5\times 9.37...\times \sin 2.4$

Approximate values are shown below for some of the lengths you may see in calculations:



dM1: Solves the problem by combining appropriate areas together which result in the total area of the concert stage (usually the sum of the areas of the **two** congruent triangles and the area of the sector).
It is dependent on the previous method marks and the B mark.

A1: awrt 214 (m^2) (condone lack of units). Must follow from a correct method. Isw if they round incorrectly.